VI. Conclusions

A configuration DSA for dynamic systems with a plate is developed by using CAD parameters selected from CAD geometry in a commercial CAD tool. With this system, the designer can easily obtain configuration design velocity fields using Pro/ENGINEER and Pro/TOOLKIT. The numerical example of a reinforcement model shows that the proposed configuration DSA of eigenvalue and frequency-response results of plate are accurate.

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Hybrid Reanalysis Method for Eigenproblems of Topological Modifications

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Introduction

STRUCTURAL optimization has been the subject of numerous studies in recent years.[1–6] Topological modifications can greatly improve a typical design; however, the solutions of topological optimization problems are difficult because of changes in the structural model. In particular, changes in the number of variables and degrees of freedom result in corresponding changes in the form of the analysis equation.

One of the main obstacles in topological modification analysis is the high computational effort involved in repeated analysis. As structural systems to be solved for static and dynamic characteristics become larger, the computing time and the corresponding cost increase drastically. Hence, various techniques have been used to reduce the size of the system or the dimensions of the structural matrices involved in the formulation.[7–12] The reduction schemes increase the calculation efficiency at the expense of solution accuracy.

In previous studies, two sets of degrees of freedom (DOF), called secondary and primary, are introduced in repeated analysis. During the solution, the secondary set is condensed out, whereas the primary one is retained. When the transformation matrix derived from the stiffness and mass matrix is used, the system to be solved is transformed into a reduced subspace represented by the primary degrees of freedom. An important problem concerns which DOF should go into the primary set. Improper selection may not only result in missing some of the lowest modes but also cause difficulties in programming because one must redecompose stiffness and mass matrices according to the selected primary and secondary sets.

Considering that the secondary and primary DOF method is used commonly in substructures, we intend to introduce this method into dynamic reanalysis of topological modification. In the present study, the DOF in the initial system are selected as the primary set whereas the ones added in the modified system are selected as the secondary set. When static condensation and Rayleigh quotient are used and the effects of the mass added in the modified system are considered, several eigenpairs are obtained simultaneously. The results show that the proposed method can give high accuracy.

Problem Formulation

We consider only the case where both the design variables and the number of DOF are added in the modified system. In this case, the generalized eigenproblem is as follows:

$$\boldsymbol{KV} = \lambda \boldsymbol{MV} \tag{1}$$

where

$$\boldsymbol{K} = \boldsymbol{K}_0' + \Delta\boldsymbol{K}' \tag{2}$$

$$\boldsymbol{M} = \boldsymbol{M}_0' + \Delta\boldsymbol{M}' \tag{3}$$

$$\boldsymbol{K}_0' = \begin{bmatrix} \boldsymbol{K}_0 & 0 \\ 0 & 0 \end{bmatrix} \tag{4}$$

$$\boldsymbol{M}_0' = \begin{bmatrix} \boldsymbol{M}_0 & 0 \\ 0 & 0 \end{bmatrix} \tag{5}$$

$$\Delta\boldsymbol{K}' = \begin{bmatrix} \Delta\boldsymbol{K}_{nn} & \Delta\boldsymbol{K}_{nm} \\ \Delta\boldsymbol{K}_{mn} & \Delta\boldsymbol{K}_{mm} \end{bmatrix} \tag{6}$$

$$\Delta\boldsymbol{M}' = \begin{bmatrix} \Delta\boldsymbol{M}_{nn} & \Delta\boldsymbol{M}_{nm} \\ \Delta\boldsymbol{M}_{mn} & \Delta\boldsymbol{M}_{mm} \end{bmatrix} \tag{7}$$

$\boldsymbol{K}_0$ and $\boldsymbol{M}_0$ are the stiffness and mass matrices of the initial structure, respectively. Subscript n denotes the number of DOF of the initial structure and m the augmentation of the DOF of the modified structure. If the DOF in the initial structure are selected as the primary set and the ones added in the modified structure are selected as the secondary one, having assembled the change of stiffness and mass matrices for the added new nodes and members, from Eqs. (1–7), it can be seen that the stiffness and mass matrices do not have to be redecomposed.

Proposed Method

Substituting Eqs. (2–7) into Eq. (1) yields

$$\begin{bmatrix} \boldsymbol{K}_0 + \Delta\boldsymbol{K}_{nn} & \Delta\boldsymbol{K}_{nm} \\ \Delta\boldsymbol{K}_{mn} & \Delta\boldsymbol{K}_{mm} \end{bmatrix} \begin{pmatrix} \boldsymbol{V}_n \\ \boldsymbol{V}_m \end{pmatrix} = \lambda \begin{bmatrix} \boldsymbol{M}_0 + \Delta\boldsymbol{M}_{nn} & \Delta\boldsymbol{M}_{nm} \\ \Delta\boldsymbol{M}_{mn} & \Delta\boldsymbol{M}_{mm} \end{bmatrix} \begin{pmatrix} \boldsymbol{V}_n \\ \boldsymbol{V}_m \end{pmatrix} \tag{8}$$

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Table 1 Comparison of eigenvalues from Fig. 2

k	Initial exact value	Modified exact value	Guyan's static condensation[7]	Relative error	Proposed method	Relative error
1	0.513067E+02	0.909199E+02	0.913709E+02	0.496047E−02	0.909238E+02	0.430464E−04
2	0.287862E+03	0.330971E+03	0.346713E+03	0.475620E−01	0.331366E+03	0.119121E−02
3	0.521675E+03	0.573782E+03	0.588422E+03	0.255145E−01	0.573173E+03	−0.106139E−02
4	0.707402E+03	0.792877E+03	0.840972E+03	0.606591E−01	0.790245E+03	−0.331879E−02
5	0.119840E+04	0.124285E+04	0.135651E+04	0.914516E−01	0.124600E+04	0.253304E−02
6	0.154818E+04	0.143572E+04	0.153446E+04	0.687746E−01	0.142804E+04	−0.535078E−02
7	0.171176E+04	0.180133E+04	0.189540E+04	0.522223E−01	0.177888E+04	−0.124631E−01
8	0.220927E+04	0.225430E+04	0.238183E+04	0.565731E−01	0.225412E+04	−0.761556E−04

Table 2 Comparison of eigenvalues from Fig. 3

k	Initial exact value	Modified exact value	Guyan's static condensation[7]	Relative error	Proposed method	Relative error
1	0.513067E+02	0.780375E+02	0.918222E+02	0.176642E+00	0.780464E+02	0.114020E−03
2	0.287862E+03	0.396425E+03	0.464513E+03	0.171754E+00	0.400915E+03	0.113252E−01
3	0.521675E+03	0.486007E+03	0.577423E+03	0.188096E+00	0.478219E+03	−0.160234E−01
4	0.707402E+03	0.906374E+03	0.106865E+04	0.179038E+00	0.907725E+03	0.149007E−02
5	0.119840E+04	0.142686E+04	0.159951E+04	0.121000E+00	0.142708E+04	0.155932E−03
6	0.154818E+04	0.144752E+04	0.169589E+04	0.171586E+00	0.144976E+04	0.154909E−02
7	0.171176E+04	0.198273E+04	0.225035E+04	0.134975E+00	0.203345E+04	0.255821E−01
8	0.220927E+04	0.224724E+04	0.242616E+04	0.796208E−01	0.229490E+04	0.212111E−01

Let the mass associated with the vector $\mathbf{V}_m$ be zero; then Eq. (8) becomes

$$\begin{bmatrix} \boldsymbol{K}_0 + \Delta\boldsymbol{K}_{nn} & \Delta\boldsymbol{K}_{nm} \\ \Delta\boldsymbol{K}_{mn} & \Delta\boldsymbol{K}_{mm} \end{bmatrix} \begin{pmatrix} \boldsymbol{V}_n \\ \boldsymbol{V}_m \end{pmatrix} = \lambda \begin{bmatrix} \boldsymbol{M}_0 + \Delta\boldsymbol{M}_{nn} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \begin{pmatrix} \boldsymbol{V}_n \\ \boldsymbol{V}_m \end{pmatrix} \tag{9}$$

From the second equation of Eq. (9), one can obtain the transformation matrix relating the vectors $\boldsymbol{V}_n$ and $\boldsymbol{V}_m$:

$$\boldsymbol{V}_m = -\Delta\boldsymbol{K}_{mm}^{-1}\Delta\boldsymbol{K}_{mn}\boldsymbol{V}_n = \boldsymbol{T}\boldsymbol{V}_n \tag{10}$$

and the eigenmode is written as

$$\boldsymbol{V} = \begin{pmatrix} \boldsymbol{V}_n \\ \boldsymbol{V}_m \end{pmatrix} = \begin{pmatrix} \boldsymbol{I} \\ \boldsymbol{T} \end{pmatrix} \boldsymbol{V}_n = \boldsymbol{S}\boldsymbol{V}_n \tag{11}$$

The preceding transformation is exact in a static sense because only the stiffness matrix is used in Eq. (10).

Substituting Eq. (11) into Eq. (1) and premultiplying Eq. (1) by $\boldsymbol{S}^T$ yields

$$\boldsymbol{S}^T\boldsymbol{K}\boldsymbol{S}\boldsymbol{V}_n = \lambda\boldsymbol{S}^T\boldsymbol{M}\boldsymbol{S}\boldsymbol{V}_n \tag{12}$$

Let

$$\boldsymbol{K}_r = \boldsymbol{S}^T\boldsymbol{K}\boldsymbol{S} \tag{13}$$

$$\boldsymbol{M}_r = \boldsymbol{S}^T\boldsymbol{M}\boldsymbol{S} \tag{14}$$

we have

$$\boldsymbol{K}_r\boldsymbol{V}_n = \lambda\boldsymbol{M}_r\boldsymbol{V}_n \tag{15}$$

Solving the condensed eigenproblem, one can obtain approximate eigenpairs of the modified structure. To improve the accuracy of the eigenpairs, when the effects of mass added in the modified structure are considered a more accurate transformation can be obtained from Eq. (8):

$$\boldsymbol{V}_m = -(\Delta\boldsymbol{K}_{mm} - \lambda\Delta\boldsymbol{M}_{mm})^{-1}(\Delta\boldsymbol{K}_{mn} - \lambda\Delta\boldsymbol{M}_{mn})\boldsymbol{V}_n \tag{16}$$

When λ and $\boldsymbol{V}_n$ derived from Eq. (15) are used, an improved $\boldsymbol{V}_m$ can be obtained from Eq. (16). With the condition of orthogonality and Rayleigh quotient, the eigenpairs obtained by the proposed method are as follows:

$$\lambda_{\text{new}} = \boldsymbol{V}^T\boldsymbol{K}\boldsymbol{V}/\boldsymbol{V}^T\boldsymbol{M}\boldsymbol{V} \tag{17}$$

$$V_{\text{new}} = \boldsymbol{V}\Big/\sqrt{\boldsymbol{V}^T\boldsymbol{M}\boldsymbol{V}} \tag{18}$$

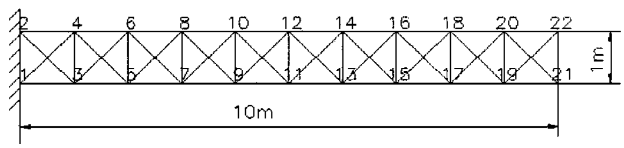

Fig. 1 Initial design of truss structure.

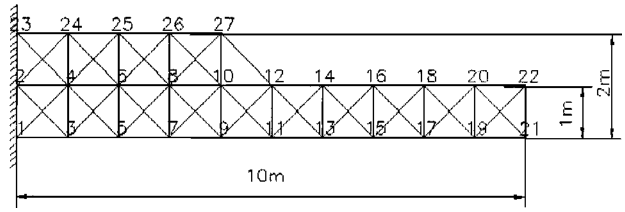

Fig. 2 Modified structure with 27 nodes and 67 members.

Numerical Examples

Consider an initial truss structure (Fig. 1) with its parameters Young's modulus $E = 2.1 \times 10^{11}$ Pa, cross-sectional area of each bar $A = 1.2 \times 10^{-4}$ m^2, and mass density $\rho = 7.8 \times 10^3$ kg/m^3.

Example 1

The topological modification with the new added 5 nodes and 17 members is shown in Fig. 2. Its parameters are the same as the initial structure. The results obtained by Guyan's static condensation[7] are compared with those obtained by the proposed method. The results are listed in Table 1.

Example 2

The topological modification with the new added 11 nodes and 40 members is shown in Fig. 3. Its parameters are also the same as the initial structure. Comparison of eigenvalues from Guyan's static condensation[7] and the proposed method is shown in Table 2.

The results show that the present method can give better approximate eigenvalues of the modified structure than Guyan's static

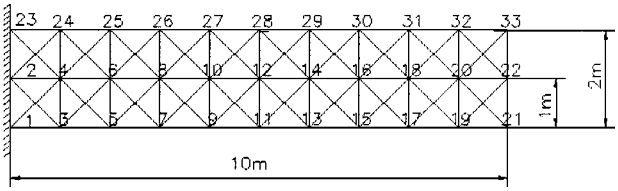

Fig. 3 Modified structure with 33 nodes and 90 members.

condensation.[7] That is because Guyan's static condensation does not contain the effects of mass from added members in the modified structure, whereas these effects are considered in the proposed method. From Eqs. (10) and (16–18), it can be seen that little computational effort is added in the proposed method. From the results, it can be also seen that the proposed method can also give high-quality accuracy for the large topological modifications.

Conclusions

A hybrid method is presented for the efficient calculation of eigenpairs of topological modifications in dynamic problems. The effects of mass from the added member are considered in this method. From Eqs. (10) and (16–18), it can be seen that the little computational effort is added in the proposed method. The results show that the proposed method is efficient for eigenproblems of topological modifications. For large topological modifications, the presented method can also give high accuracy.

Acknowledgments

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Toward More Effective Genetic Algorithms for the Optimization of Piezoelectric Actuator Locations

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Introduction

BECAUSE of the well-known advantages of adaptive structures over traditional structures, significant research is being conducted on these structures at present (for example, Ref. 1). One key requirement toward the success of adaptive structures would be in choosing the optimal location of actuators. A large number of publications[2–5] have addressed this problem. In our previous paper[5] various methods addressing this issue were mentioned. Among various approaches being used, genetic algorithms (GA) were found to be popular. Some advantages and disadvantages of GAs were presented in our previous paper.[5] In previous studies the present authors successfully applied two versions of GAs (termed GA version 1 and GA version 2). Both of the versions were adapted from Carroll's FORTRAN Genetic Algorithm Driver, to solve two kinds of difficult, computationally intensive, combinatorial, and continuous large-scale optimization problems [code available online at http://www.aic.nrl.navy.mil:80/galist/src/#fortran (cited 3 May 2002)]. These problems include finding both an optimal placement and optimal voltages of 30 piezoelectric actuators, from 193 candidate locations, with more than 1.28×10^{35} possible solutions to obtain the best correction to the surface thermal distortions of a thin hexagonal spherical primary mirror (Fig. 1a in Ref. 5) of an astronomical telescope. The thermal distortions were caused by four different types of spatial temperature distributions. The two types of optimization problems were as follows: 1) to find the optimal locations and optimal voltages suitable for each type of thermal loads individually and 2) to determine just one set of actuator locations that would reduce the distortion caused by all four types of thermal loads. The latter problem is a more challenging, multicriterion optimization problem. A laminated triangular shell element[6] was used to model the mirror. The main conclusions from our previous studies are as follows: 1) the design search space is highly multimodal; 2) both GA version 1 and GA version 2 are effective for the optimization of piezoelectric actuator locations; 3) GA version 2 has more flexibility than GA version 1; 4) GA version 2 can get modestly better results than DeLorenzo algorithm for both optimization problems for the case of 30 piezoelectric actuators; 5) the convergence to a solution can occur without reaching an optimal or near-optimal solution; 6) more than one suboptimal solution to each problem was found; 7) optimal location obtained for one type of thermal loads may perform poorly for other types of thermal loads; and 8) GAs can determine one set of actuator locations, which is good for all four of the types of thermal loads considered for these studies. The needed voltages will be different for different thermal loads.

In the present Note an improved GA, termed GA version 3 and adapted from the GA version 2, is employed to resolve the two

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